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LETTER TO THE EDITOR

Magnon-mediated superconductivity in itinerant ferromagnets

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Abstract

This letter discusses magnon-mediated superconductivity in ferromagnetic metals. The mechanism explains in a natural way the fact that the superconductivity in UGe_2 , ZrZn_2 and URhGe is apparently confined to the ferromagnetic phase. The order parameter is a spin anti-parallel component of a spin-1 triplet with zero spin projection. The transverse spin fluctuations are pair forming and the longitudinal ones are pair breaking. The competition between magnons and paramagnons explains the existence of two successive quantum phase transitions in UGe_2 , from ferromagnetism to ferromagnetic superconductivity, and at higher pressure to paramagnetism. The maximum T_{SC} results from the suppression of the paramagnon contribution. To form a Cooper pair an electron transfers from one Fermi surface to the other. As a result, the onset of superconductivity leads to the appearance of two Fermi surfaces in each of the spin up and spin down momentum distribution functions. This fact explains the linear temperature dependence at low temperature of the specific heat, and the experimental results for UGe_2 .

(Some figures in this article are in colour only in the electronic version)

The discovery of superconductivity in single crystals of UGe_2 [1, 2], ZrZn_2 [3] and URhGe [4] revived the interest in the coexistence of superconductivity and ferromagnetism. The experiments indicate that in very pure systems, and at very low temperature, ferromagnetism and superconductivity can coexist, with the same electrons that cause the magnetism also responsible for the superconductivity. The superconductivity is apparently confined to the ferromagnetic phase. There are two successive quantum phase transitions in UGe_2 [1, 2], from ferromagnetism to ferromagnetic superconductivity, and at higher pressure to paramagnetism. The specific heat anomaly associated with the superconductivity appears to be absent and the specific heat depends linearly on the temperature at low temperature [3–5].

The paramagnon-mediated superconductivity [6, 7] has long been considered the most promising theory of coexistence of superconductivity and ferromagnetism. The order parameters are spin parallel components of the spin triplet. The superconductivity in ZrZn_2 was predicted. Nevertheless, the theory meets some difficulties. It predicts that the superconductivity occurs in both the ferromagnetic and the paramagnetic phases. A solution of the problem was recently proposed. It has been shown [8] that the critical temperature is much higher in the ferromagnetic phase than in the paramagnetic one due to the coupling of the magnons to the longitudinal spin fluctuations. Alternatively, the superconductivity in the ferromagnetic state of ZrZn_2 is explained in [9] as a result of an exchange-type interaction between the magnetic moments of triplet-state Cooper pairs and the ferromagnetic magnetization density in the Ginzburg–Landau mean field theory. The Fay and Apple (FA) theory predicts that spin up and spin down fermions form Cooper pairs, and hence the specific heat decreases exponentially at low temperature. The phenomenological theories [10, 11] circumvent the problem, assuming that only majority spin fermions form pairs, and hence the minority spin fermions contribute the asymptotic of the specific heat. The coefficient $\gamma = \frac{C}{T}$ is half the size in the superconducting phase, which closely matches the experiments with URhGe [4]. But the assumption is quite questionable when the magnetization approaches zero. The superconducting critical temperature in FA theory increases when the magnetization decreases and very close to the quantum critical point falls rapidly. It has recently been the subject of controversial debate. It is obtained in [12], by means of a more complete Eliashberg treatment, that the transition temperature is nonzero at the critical point. In [13], however, the authors have shown that the reduction of quasiparticle coherence and life-time due to spin fluctuations is the pair-breaking process which leads to a rapid reduction of the superconducting critical temperature near the quantum critical point. Finally, recent studies of polycrystalline samples of UGe_2 show that the T – P phase diagram is very similar to those of single-crystal specimens of UGe_2 [14]. These findings suggest that the superconductivity in UGe_2 is relatively insensitive to the presence of impurities and defects which excludes the spin parallel pairing.

In the present letter an itinerant system is considered in which the spin- $\frac{1}{2}$ fermions responsible for the ferromagnetism are the same quasiparticles which form the Cooper pairs. The interaction of quasiparticles $c_\sigma(\vec{x})(c_\sigma^\dagger(\vec{x}))$ with spin fluctuations has the form

$$H_{s-f} = J \int d^3x c^\dagger(\vec{x}) \frac{\vec{\tau}}{2} c(\vec{x}) \cdot \vec{M}(\vec{x}) \quad (1)$$

where the transverse spin fluctuations are described by magnons $M_1(\vec{x}) + iM_2(\vec{x}) = \sqrt{2M}a(\vec{x})$, $M_1(\vec{x}) - iM_2(\vec{x}) = \sqrt{2M}a^+(\vec{x})$ and the longitudinal spin fluctuations by paramagnons $M_3(\vec{x}) - M = \varphi(\vec{x})$. M is the zero-temperature dimensionless magnetization of the system per lattice site. The magnon's dispersion is $\omega(\vec{k}) = \rho\vec{k}^2$ where the spin stiffness constant is proportional to M ($\rho = M\rho_0$), and the paramagnon propagator is [15, 16]

$$D_{pm} = \frac{1}{r - i\frac{\omega}{|\vec{p}|} + b\vec{p}^2}. \quad (2)$$

The parameter r is the inverse static longitudinal magnetic susceptibility, which measures the deviation from the quantum critical point. The constants J , ρ_0 and b are phenomenological ones subject to some relations.

Integrating out the spin fluctuations, one obtains an effective four-fermion theory which can be written as a sum of four terms. Three of them describe the interaction of the components of spin-1 composite fields ($\uparrow\uparrow$, $\uparrow\downarrow + \downarrow\uparrow$, $\downarrow\downarrow$) which have a projection of spin 1, 0 and -1 respectively. The fourth term describes the interaction of the spin singlet composite fields

$\uparrow\downarrow - \downarrow\uparrow$. The spin singlet fields' interaction is repulsive and does not contribute to the superconductivity [17]. The spin parallel fields' interactions are due to the exchange of paramagnons and do not contribute to the magnon-mediated superconductivity. The relevant interaction is that of the $\uparrow\downarrow + \downarrow\uparrow$ fields. In the static approximation, the Hamiltonian of interaction is

$$H_{int} = -\frac{J^2}{16} \int \prod_i \frac{d^3k_i}{(2\pi)^3} [c_{\uparrow}^+(\vec{k}_1)c_{\downarrow}^+(\vec{k}_2) + c_{\downarrow}^+(\vec{k}_1)c_{\uparrow}^+(\vec{k}_2)] \times [c_{\uparrow}(\vec{k}_2 - \vec{k}_3)c_{\downarrow}(\vec{k}_1 + \vec{k}_3) + c_{\downarrow}(\vec{k}_2 - \vec{k}_3)c_{\uparrow}(\vec{k}_1 + \vec{k}_3)]V(\vec{k}_3) \quad (3)$$

where the potential

$$V(\vec{k}) = \frac{2M}{\rho k^2} - \frac{1}{r + bk^2} \quad (4)$$

has an attracting part due to exchange of magnons and a repulsive part due to exchange of paramagnons. The effective Hamiltonian of the system is

$$H_{eff} = H_0 + H_{int} \quad (5)$$

where H_0 is the Hamiltonian of the free spin up and spin down fermions with dispersions

$$\epsilon_{\uparrow}(\vec{k}) = \frac{\vec{k}^2}{2m} - \mu - \frac{JM}{2}, \quad \epsilon_{\downarrow}(\vec{k}) = \frac{\vec{k}^2}{2m} - \mu + \frac{JM}{2}. \quad (6)$$

One can obtain the gap equation following the standard technique. To ensure that the fermions which form Cooper pairs are the same as those responsible for spontaneous magnetization, one has to consider the equation for the magnetization

$$M = \frac{1}{2} \langle c_{\uparrow}^{\dagger} c_{\uparrow} - c_{\downarrow}^{\dagger} c_{\downarrow} \rangle \quad (7)$$

as well. Then the system of equations for the gap and for the magnetization determines the phase where the superconductivity and the ferromagnetism coexist. The system can be written in terms of Bogoliubov excitations, which have the following dispersion relations:

$$E_1(\vec{k}) = -\frac{JM}{2} - \sqrt{\epsilon^2(\vec{k}) + |\Delta(\vec{k})|^2} \quad E_2(\vec{k}) = \frac{JM}{2} - \sqrt{\epsilon^2(\vec{k}) + |\Delta(\vec{k})|^2} \quad (8)$$

where $\Delta(\vec{k})$ is the gap, and $\epsilon(\vec{k}) = \frac{\vec{k}^2}{2m} - \mu$.

At zero temperature the equations take the form

$$M = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} [1 - \Theta(-E_2(\vec{k}))] \quad (9)$$

$$\Delta(\vec{p}) = \frac{J^2}{8} \int \frac{d^3k}{(2\pi)^3} \frac{V(\vec{p} - \vec{k})\Theta(-E_2(\vec{k}))}{\sqrt{\epsilon^2(\vec{k}) + |\Delta(\vec{k})|^2}} \Delta(\vec{k}). \quad (10)$$

The gap is an antisymmetric function $\Delta(-\vec{k}) = -\Delta(\vec{k})$, so that the expansion in terms of spherical harmonics $Y_{lm}(\Omega_{\vec{k}})$ contains only terms with odd l . I assume that the component with $l = 1$ and $m = 0$ is nonzero and the other ones are zero

$$\Delta(\vec{k}) = \Delta_{10}(k) \sqrt{\frac{3}{4\pi}} \cos \theta. \quad (11)$$

Expanding the potential in terms of Legendre polynomial P_l one obtains that only the component with $l = 1$ contributes the gap equation. The potential $V_1(p, k)$ has the form

$$V_1(p, k) = \frac{3M}{\rho} \left[\frac{p^2 + k^2}{4p^2k^2} \ln \left(\frac{p+k}{p-k} \right)^2 - \frac{1}{pk} \right] - \frac{3M}{\rho} \beta \left[\frac{p^2 + k^2}{4p^2k^2} \ln \frac{r' + (p+k)^2}{r' + (p-k)^2} - \frac{1}{pk} \right], \quad (12)$$

where $\frac{3M}{\rho} = \frac{3}{\rho_0}$, $\beta = \frac{\rho}{2Mb} = \frac{\rho_0}{2b} > 1$ and $r' = \frac{r}{b} \ll 1$. A straightforward analysis shows that for a fixed p the potential is positive when k runs over an interval around $p(p - \Lambda, p + \Lambda)$, where Λ is approximately independent of p . In order to allow for an explicit analytic solution, I introduce further simplifying assumptions by neglecting the dependence of $\Delta_{10}(k)$ on k ($\Delta_{10}(k) = \Delta_{10}(p_f) = \Delta$) and setting $V_1(p_f, k)$ equal to a constant V_1 within the interval $(p_f - \Lambda, p_f + \Lambda)$ and zero elsewhere. The system of equations (9), (10) is then reduced to the system

$$M = \frac{1}{8\pi^2} \int_0^\infty dk k^2 \int_{-1}^1 dt [1 - \Theta(-E_2(k, t))] \quad (13)$$

$$\Delta = \frac{J^2 V_1}{32\pi^2} \int_{p_f - \Lambda}^{p_f + \Lambda} dk k^2 \int_{-1}^1 dt t^2 \frac{\Theta(-E_2(k, t))}{\sqrt{\epsilon^2(k) + \frac{3}{4\pi} t^2 \Delta^2}} \Delta \quad (14)$$

where $t = \cos \theta$.

One looks for a solution of the system which satisfies $\sqrt{\frac{3}{\pi}} \Delta < JM$. Then the equation $E_2(k, t) = 0$ defines the Fermi surfaces

$$k_f^\pm = \sqrt{p_f^2 \pm m \sqrt{J^2 M^2 - \frac{3}{\pi} t^2 \Delta^2}}, \quad p_f = \sqrt{2\mu m}. \quad (15)$$

The domain between the Fermi surfaces contributes to the magnetization M in equation (13), but it is cut out from the domain of integration in the gap equation (14). One is primarily interested in determining at what magnetization a superconductivity exists. When the magnetization increases, the domain of integration in the gap equation decreases. Near the quantum critical point the size of the gap is small, and hence the linearized gap equation can be considered. Then it is easy to obtain the critical value of the magnetization M_{SC}

$$M_{SC} = \frac{4p_f^2}{mJ} \left(1 + \frac{4p_f^2 - \Lambda^2}{\Lambda^2} \exp \left[\frac{64\pi^2}{3J^2 V_1 m p_f} \right] \right)^{-\frac{1}{2}}. \quad (16)$$

Near the second quantum phase transition, when the magnetization approaches zero, the domain between the Fermi surfaces decreases. One can approximate the equation for magnetization equation (13) substituting k_f^\pm from equation (15) in the the difference $(k_f^+)^2 - (k_f^-)^2$ and setting $k_f^\pm = p_f$ elsewhere. Then, in this approximation, the magnetization is linear in Δ , namely

$$\Delta = \sqrt{\frac{\pi}{3}} J \kappa M \quad (17)$$

where κ runs the interval (0, 1), and satisfies the equation

$$\kappa \sqrt{1 - \kappa^2} + \arcsin \kappa = \frac{8\pi^2}{m p_f J}. \quad (18)$$

The equation (18) has a solution if $m p_f J > 16\pi$. Substituting M from equation (17) in equation (14), one arrives at an equation for the gap. This equation can be solved in a standard way and the solution is

$$\Delta = \sqrt{\frac{16\pi}{3} \frac{\Lambda p_f \kappa}{m}} \exp \left[-\frac{3}{2} I(\kappa) - \frac{24\pi^2}{J^2 V_1 m p_f} \right] \quad (19)$$

$$I(\kappa) = \int_{-1}^1 dt t^2 \ln(1 + \sqrt{1 - \kappa^2 t^2}).$$

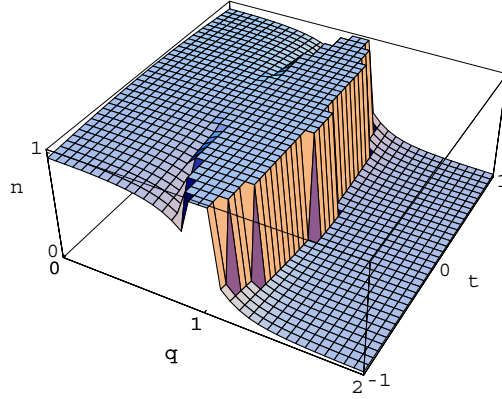


Figure 1. The zero-temperature momentum distribution n , for spin up fermions, as a function of $q = \frac{p}{p_f}$ and $t = \cos \theta$.

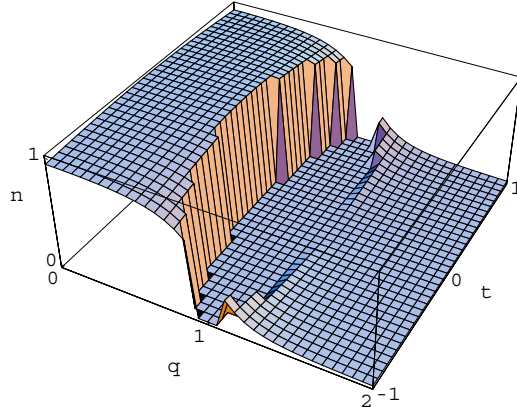


Figure 2. The zero-temperature momentum distribution n , for spin down fermions, as a function of $q = \frac{p}{p_f}$ and $t = \cos \theta$.

Equations (17)–(19) are the solution of the system of equations (13) and (14) near the quantum transition to paramagnetism.

When superconductivity and ferromagnetism coexist, the momentum distribution functions $n^\uparrow(p, t)$ and $n^\downarrow(p, t)$ of the spin up and spin down quasiparticles have two Fermi surfaces each. One can write them in terms of the distribution functions of the Bogoliubov fermions

$$\begin{aligned} n^\uparrow(p, t) &= u^2(p, t)n_1(p, t) + v^2(p, t)n_2(p, t) \\ n^\downarrow(p, t) &= u^2(p, t)(1 - n_1(p, t)) + v^2(p, t)(1 - n_2(p, t)) \end{aligned} \quad (20)$$

where $u(p, t)$ and $v(p, t)$ are the coefficients in the Bogoliubov transformation. At zero temperature $n_1(p, t) = 1$, $n_2(p, t) = \Theta(-E_2(p, t))$, and the Fermi surfaces (15) manifest themselves both in the spin up and spin down momentum distribution functions. The functions are depicted in figures 1 and 2.

The two Fermi surfaces are necessary for the existence of itinerant ferromagnetism, and explain the mechanism of Cooper pairing. In the ferromagnetic phase n^\uparrow and n^\downarrow have different (majority and minority) Fermi surfaces. To form a spin anti-parallel Cooper pair, the fermion

has to transfer from one Fermi surface to the other. If the value of the momentum of the emitted or absorbed magnon lies in the domain where the effective potential is attracting but outside the domain between the two Fermi surfaces, fermions with opposite spins form a Cooper pair. As a result, the onset of superconductivity is accompanied by the appearance of a second Fermi surface in each of the spin up and spin down momentum distribution functions.

The existence of the two Fermi surfaces also explains the linear dependence of the specific heat at low temperatures as opposed to the exponential decrease of the specific heat in the BCS theory:

$$\frac{C}{T} = \frac{2\pi^2}{3}(N^+(0) + N^-(0)). \quad (21)$$

Here $N^\pm(0)$ are the densities of states on the Fermi surfaces. One can rewrite the $\gamma = \frac{C}{T}$ constant in terms of an elliptic integral of the second kind $E(\alpha, x)$

$$\gamma = \frac{mp_f}{3\kappa} \left[(1+s)^{\frac{1}{2}} E\left(\frac{1}{2} \arcsin \kappa, \frac{2s}{s+1}\right) + (1-s)^{\frac{1}{2}} E\left(\frac{1}{2} \arcsin \kappa, \frac{2s}{s-1}\right) \right] \quad (22)$$

where $s = \frac{JMm}{p_f^2} < 1$ and $\kappa = \sqrt{\frac{3}{\pi} \frac{\Delta}{JM}}$. The equation (22) shows that in the ferromagnetic phase ($\Delta = 0$) the specific heat constant γ is smaller than in the superconducting one, which closely matches the experiments with UGe₂ [5]. An important point is that UGe₂ is an anisotropic ferromagnet and hence the magnon has a gap which changes the potential equation (12). The physical consequence of the change is that the superconductivity disappears before the quantum phase transition from ferromagnetism to paramagnetism (see [5, 14]). The distance between these two points depends on the anisotropy.

The presence of an additional phase line T_x and the correlation between a transition at T_x and the appearance of superconductivity in UGe₂ has been proved [2, 5]. It lies entirely within the ferromagnetic phase and is suggested by a strong anomaly in the resistivity at a temperature T_x . The maximum transition temperature for superconductivity is near the pressure P_x where T_x vanishes. The authors have assumed that superconductivity is mediated by fluctuations associated with a (hypothetical) second order critical point P_x , with an unidentified order parameter.

Experimental measurement of ac magnetic susceptibility as a function of the temperature indicates a peak at the ferromagnetic Curie temperature, as usual. But the peak is substantially damped at a pressure near the maximal superconducting critical temperature [18]. The suppression of the peak can be understood as a suppression of the paramagnon contribution, which in turn means suppression of pair breaking effects and hence higher superconducting critical temperature. So the proposed model of magnon-mediated superconductivity complemented by the experimental results explains, at least qualitatively, the superconductivity in UGe₂ without invoking an additional phase transition.

The proposed model of ferromagnetic superconductivity differs from the models discussed in [6–11] in two ways. First, the superconductivity is due to the exchange of magnons, and paramagnons have a pair breaking effect. Second, the order parameter is a spin anti-parallel component of a spin triplet with zero spin projection. The existence of two Fermi surfaces in each of the spin up and spin down momentum distribution functions is a generic property of a ferromagnetic superconductivity with spin anti-parallel pairing (see also [19]). They lead to a linear temperature dependence of the specific heat at low temperature. But the experimental result has an alternative theoretical explanation in [4, 10, 11]. So, one needs an experiment which proves undoubtedly the existence of the predicted Fermi surfaces.

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